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3 (Sem-1/CBCS) MAT HC 1

2021

(Held in 2022)

MATHEMATICS

(Honours)

Paper : MAT-HC-1016

(Calculus)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$
 - (a) Write down the n th derivative of $y = \log x$.
 - (b) The point $P(c, f(c))$ on the graph of $f(x)$ is such that $f''(c) = 0$. Does it necessarily imply that P is an inflection point on the graph ?

Contd.

(c) Write down the value of $\lim_{x \rightarrow +\infty} x \sin \frac{1}{x}$.

(d) Find the domain of the vector function

$$\vec{F}(t) = (1-t)\hat{i} + \sqrt{-t}\hat{j} + \frac{1}{t-2}\hat{k}$$

(e) Write one basic difference between the disk/washer and shell method for computing volume of revolution.

(f) What is the direction of velocity of a moving object on its trajectory.

(g) The velocity of a particle moving in space is $\vec{v}(t) = e^t \hat{i} + t^2 \hat{j}$. Find the direction of motion at time $t=2$.

Answer the following questions : $2 \times 4 = 8$

(a) Applying L.Hopital's rule, evaluate

$$\lim_{x \rightarrow \pi/4} (1 - \tan x) \sec 2x$$

(b) Write down the parametric equation of a line that contains the point $(3,1,4)$ and is parallel to the vector $\vec{v} = -\hat{i} + \hat{j} - 2\hat{k}$.

(c) Find the area of the surface generated by revolving the portion of the curve $y = x^3$ between $x=0$ and $x=1$ about the x -axis.

(d) Explain briefly why the acceleration of an object moving with constant speed is always orthogonal to the direction of motion.

3. Answer **any three** of the following questions :

$5 \times 3 = 15$

(a) If $y = \cos(m \sin^{-1} x)$, show that

$$(1-x^2)y_{n+2} - (2n+1)x y_{n+1} + (m^2 - n^2)y_n = 0.$$

Hence find $y_n(0)$.

$3+2=5$

(b) Sketch the graph of a function f with all the following properties : 5

(i) the graph has $y=1$ and $x=3$ as asymptotes

(ii) f is increasing for $x < 3$ and $3 < x < 5$ and decreasing elsewhere

(iii) the graph is concave up for $x < 3$ and concave down for $3 < x < 7$

(iv) $f(0) = 4 = f(5)$ and $f(7) = 2$

(c) Sketch the graph of $y = \frac{3x-5}{x-2}$ identifying the locations of intercepts, concavity and inflection points (if any) and asymptotes. 5

(d) Obtain the reduction formula for $\int \tan^n x dx$.

Hence evaluate $\int_0^{\pi/4} \tan^5 x dx$ 3+2=5

(e) The position vector of a moving object at any time t is given by $\vec{R}(t) = t\hat{i} + e^t\hat{j}$. Find the tangential and normal components of the object's acceleration. 5

Answer **any three** of the following questions : 10×3=30

(a) A firm determines that x units of its product can be sold daily at rupees p per unit where $x=1000-p$. The cost of producing x units per day is

$C(x) = 3000 + 20x$. Then —

(i) Find the revenue function $R(x)$. 2

(ii) Find the profit function $p(x)$. 2

(iii) Assuming that production capacity is at most 500 units per day, determine how many units the company must produce and sell each day to maximize profit. 3

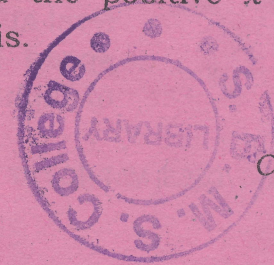
(iv) Find the maximum profit. 2

(v) What price per unit must be charged to obtain maximum profit? 1

(b) When is an object said to move in central force field? Derive Kepler's 2nd law of motion, assuming that planetary motion occurs in central force field. 2+8=10

(c) (i) Find the length of the arc of the astroid $x^{2/3} + y^{2/3} = 1$ lying in the positive quadrant. 3

(ii) Using cylindrical shell method, find the volume of the solid formed by revolving the region bounded by the parabola $y = 1 - x^2$, the y -axis, and the positive x -axis, about y -axis. 4



- (iii) Find the surface area generated when the polar curve

$$r = 5, 0 \leq \theta \leq \pi/3$$

is revolved about x -axis. 3

- (d) (i) Find the volume generated by disk/washer method, when the region bounded by $y = x$, $y = 2x$ and $y = 1$ is revolved about the x -axis 5

- (ii) A particle moves along the polar path (r, θ) where

$$r(t) = 3 + 2 \sin t, \theta(t) = t^3.$$

Find the velocity $\vec{v}(t)$ and acceleration $\vec{A}(t)$ in terms \hat{u}_r and \hat{u}_θ . 5

- (e) (i) Evaluate $\lim_{x \rightarrow 0} (1 + \sin x)^{1/x}$. 3

- (ii) Examine the existence of vertical tangent and cusp of the graph of $y = (x - 4)^{2/3}$. 3

- (iii) A projectile is fired from ground level at an angle of 30° with muzzle speed 110 ft/sec . Find the time of flight and the range. 4

- (f) (i) Obtain the reduction formula for
 $\int \cos^n x \, dx.$

Hence evaluate $\int \cos^5 x \, dx.$

$$3+2=5$$

- (ii) Find the unit tangent vector $\vec{T}(t)$
and principal unit normal vector
 $\vec{N}(t)$ at each point on the graph of
vector function

$$\vec{R}(t) = (3 \sin t, 4t, 3 \cos t) \quad 5$$

